Name solutions

October 26, 2011

ECE 311 Exam 2 Fall 2011

Closed Text and Notes

1) Be sure you have 10 pages.

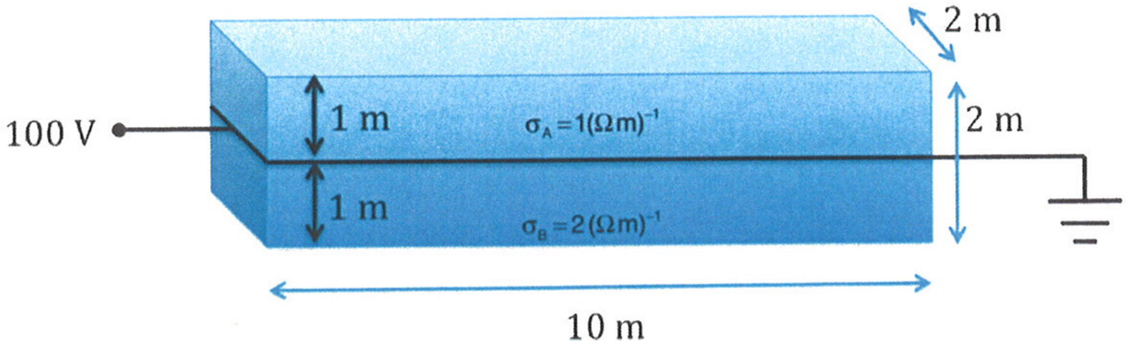
2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) no calculators allowed

4) Write neatly, if your writing is illegible then print.

5) This exam is worth 100 points.

(10 pts) 1. A conductor consists of two materials A and B. Material A has conductivity $\sigma_A = 1(\Omega m)^{-1}$ and material B $\sigma_B = 2(\Omega m)^{-1}$. The conductor is of length 10 m and a potential of 100 V is placed across the length of the conductor. The area of the cross-section of the conductor is $4m^2$ and half the conductor is of material A and half of Material B as shown.



(4 pts) What is the electric field in material A and B?

$$E_A = E_B = \frac{100V}{10m} = 10\frac{V}{m}$$

(2 pts) What is the current density in material A and B?

$$J_A = \sigma_A E_A = 1(\Omega m)^{-1}\, 10\frac{V}{m} = 10\frac{A}{m^2}$$

$$J_B = \sigma_B E_A = 2(\Omega m)^{-1}\, 10\frac{V}{m} = 20\frac{A}{m^2}$$

(2 pts) What is the total current flowing?

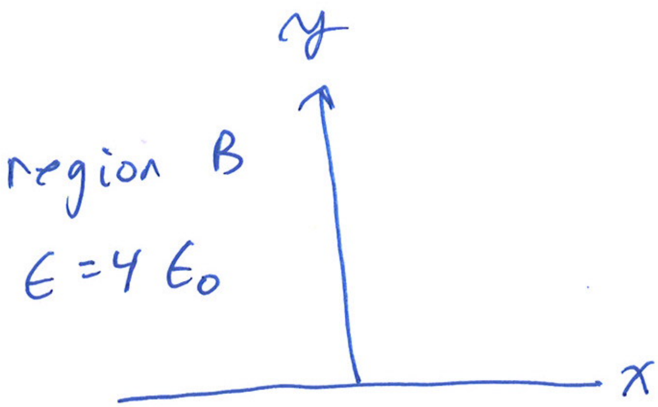
$$I = J_A(2m^2) + J_B(2m^2) = \left(10\frac{A}{m^2}\right)(2m^2) + \left(20\frac{A}{m^2}\right)(2m^2)$$

$$I = 20A + 40A = 60A$$

(2 pts) What is the resistance of the conductor?

$$R = \frac{V}{I} = \frac{100V}{60A} = \frac{5}{3}\Omega$$

(10 pts) 2. For region A, y < 0, $\varepsilon = 2\varepsilon_0$ and $\mathbf{D} = 8\,\hat{\mathbf{a}}_x + 12\,\hat{\mathbf{a}}_y + 4\,\hat{\mathbf{a}}_z\ \frac{\text{C}}{\text{m}^2}$. Find **D** in region B, where y > 0 and $\varepsilon = 4\varepsilon_0$.



Region A
$\epsilon = 2\epsilon_0$

$$\vec{D}_{nA} = 12\,\hat{a}_y\ \frac{C}{m^2}$$

$$\vec{D}_{TA} = 8\hat{a}_x + 4\hat{a}_z\ \frac{C}{m^2}$$

$$\vec{E}_{TA} = \frac{\vec{D}_{TA}}{2\epsilon_0} = \frac{1}{\epsilon_0}\left(4\hat{a}_x + 2\hat{a}_z\right)\frac{C}{m^2}$$

$$\vec{D}_{nB} = \vec{D}_{nA} = 12\,\hat{a}_y\ \frac{C}{m^2}$$

$$\vec{E}_{TB} = \vec{E}_{TA} = \frac{1}{\epsilon_0}\left(4\hat{a}_x + 2\hat{a}_z\right)\ \frac{C}{m^2}$$

$$\vec{D}_{TB} = 4\epsilon_0\,\vec{E}_{TB} = 16\hat{a}_x + 8\hat{a}_z\ \frac{C}{m^2}$$

$$\vec{D}_B = 16\hat{a}_x + 12\hat{a}_y + 8\hat{a}_z\ \frac{C}{m^2}$$

(6 pts) 3. A dielectric consists of $10^{22} \frac{\text{non-polar atoms}}{\text{m}^3}$. When the electric field $\mathbf{E} = 10^4 \frac{\text{V}}{\text{m}} \hat{\mathbf{a}}_x$ is applied to this dielectric, all the atoms can be modeled as electric dipoles with electric dipole moment $\mathbf{p} = 10^{-19}$ Cm $\hat{\mathbf{a}}_x$. What is the resulting Polarization of the dielectric?

$$\vec{P} = \frac{\text{elec. dipole moment}}{\text{unit volume}} = n\vec{p}$$

$$= \left(10^{22} \frac{\text{atoms}}{\text{m}^3}\right)\left(10^{-19} \frac{\text{cm}}{\text{atom}} \hat{a}_x\right)$$

$$= 10^3 \frac{\text{C}}{\text{m}^2} \hat{a}_x$$

(10 pts) 4. An air-filled parallel plate capacitor of capacitance 1F has a charge placed on it such that the potential between the plates is 1V. Without disturbing the charge on the plates, the plates are pulled apart to twice the separation.

(5 pts) What is the potential between the plates?

The electric field will remain the same between the plates. So with this electric field across twice the distance the potential will double 2V

(5 pts) What is the energy stored in the capacitor?

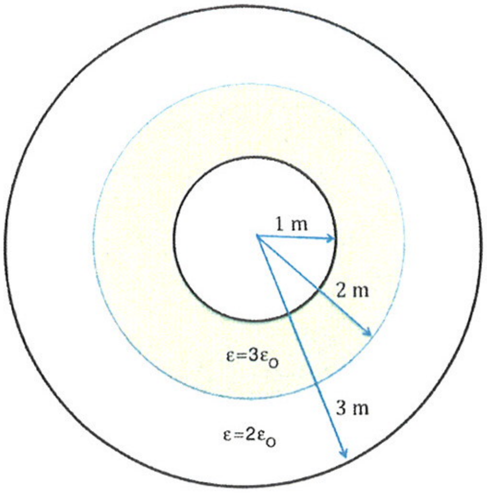
The capacitance will become 0.5F by doubling the plate spacing.

so $W = \frac{1}{2}CV^2 = \frac{1}{2}(0.5F)(2V)^2 = 1FV^2$

$$= 1\frac{C}{V}V^2 = 1\,CV = 1\,C\frac{J}{C}$$

$$= 1\,J$$

(12 pts) 5. A capacitor consists of two co-centric hollow metal spheres. The radius of one sphere is 1m and the radius of the other sphere is 3m. There are two different dielectric materials between the spheres. For 1m < r < 2 m $\varepsilon = 3\varepsilon_O$ and for 2m < r < 3 m $\varepsilon = 2\varepsilon_O$ Determine the capacitance.



place a charge of +Q on the inner sphere and -Q on the outer sphere.

so, $\vec{D} = \frac{Q}{4\pi r^2}\hat{a}_r$, $1m < r < 3m$

$$\vec{E} = \frac{Q}{12\pi\epsilon_0 r^2}\hat{a}_r \qquad 1m < r < 2m$$

$$= \frac{Q}{8\pi\epsilon_0 r^2}\hat{a}_r \qquad 2m < r < 3m$$

$$V = -\int_{3m}^{1m}\vec{E}\cdot d\vec{l} = -\int_{3m}^{2m}\frac{Q}{8\pi\epsilon_0 r^2}dr - \int_{2m}^{1m}\frac{Q}{12\pi\epsilon_0 r^2}dr$$

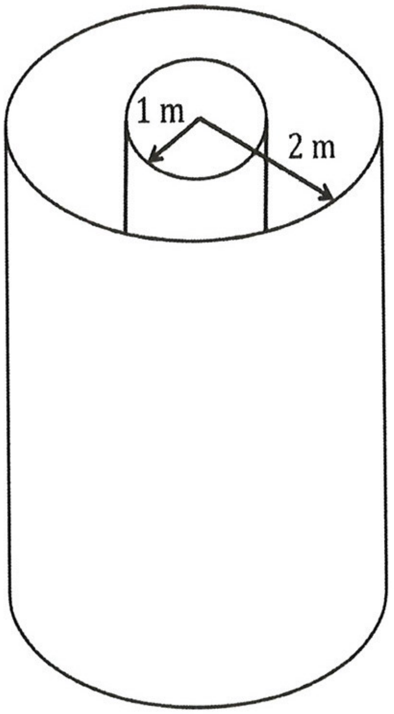
$$= \frac{Q}{8\pi\epsilon_0}\left(\frac{1}{r}\Big|_{3m}^{2m}\right) + \frac{Q}{12\pi\epsilon_0}\left(\frac{1}{r}\Big|_{2m}^{1m}\right)$$

$$= \frac{Q}{8\pi\epsilon_0}\left(\frac{1}{2m} - \frac{1}{3m}\right) + \frac{Q}{12\pi\epsilon_0}\left(\frac{1}{1m} - \frac{1}{2m}\right)$$

$$= \frac{Q}{8\pi\epsilon_0}\frac{1}{6m} + \frac{Q}{12\pi\epsilon_0}\frac{1}{2m} = \frac{Q}{48\pi\epsilon_0 m} + \frac{Q}{24\pi\epsilon_0 m}$$

$$= \frac{3Q}{48\pi\epsilon_0 m} \qquad C = \frac{Q}{V} = \frac{48\pi\epsilon_0 m}{3} = 16\pi\epsilon_0 m$$

(10 pts) 6. Two co-centric hollow infinitely long cylinders are centered on the z-axis. The cylinders have radii of $\rho=1\text{m}$ and $\rho=2\text{m}$. A voltage is applied across the cylinders such that $V(\rho=1\text{m}) = 0$ and $V(\rho=2\text{m}) = 100\text{V}$. Determine the potential between the cylinders.



$$\nabla^2 V = 0 \qquad V(1\text{m}) = 0$$

$$V(2\text{m}) = 100\text{V}$$

Cylindrical symmetry and V will only vary with ρ.

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \underbrace{\frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2}}_{=0} + \underbrace{\frac{\partial^2 V}{\partial z^2}}_{=0} = 0$$

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) = 0$$

$$\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) = 0$$

$$\rho \frac{\partial V}{\partial \rho} = A$$

$$\partial V = A \frac{\partial \rho}{\rho}$$

$$V = A \ln \rho + B$$

$$V(\rho=1) = A \ln(1) + B = 0$$

$$= 0 + B = 0$$

$$B = 0$$

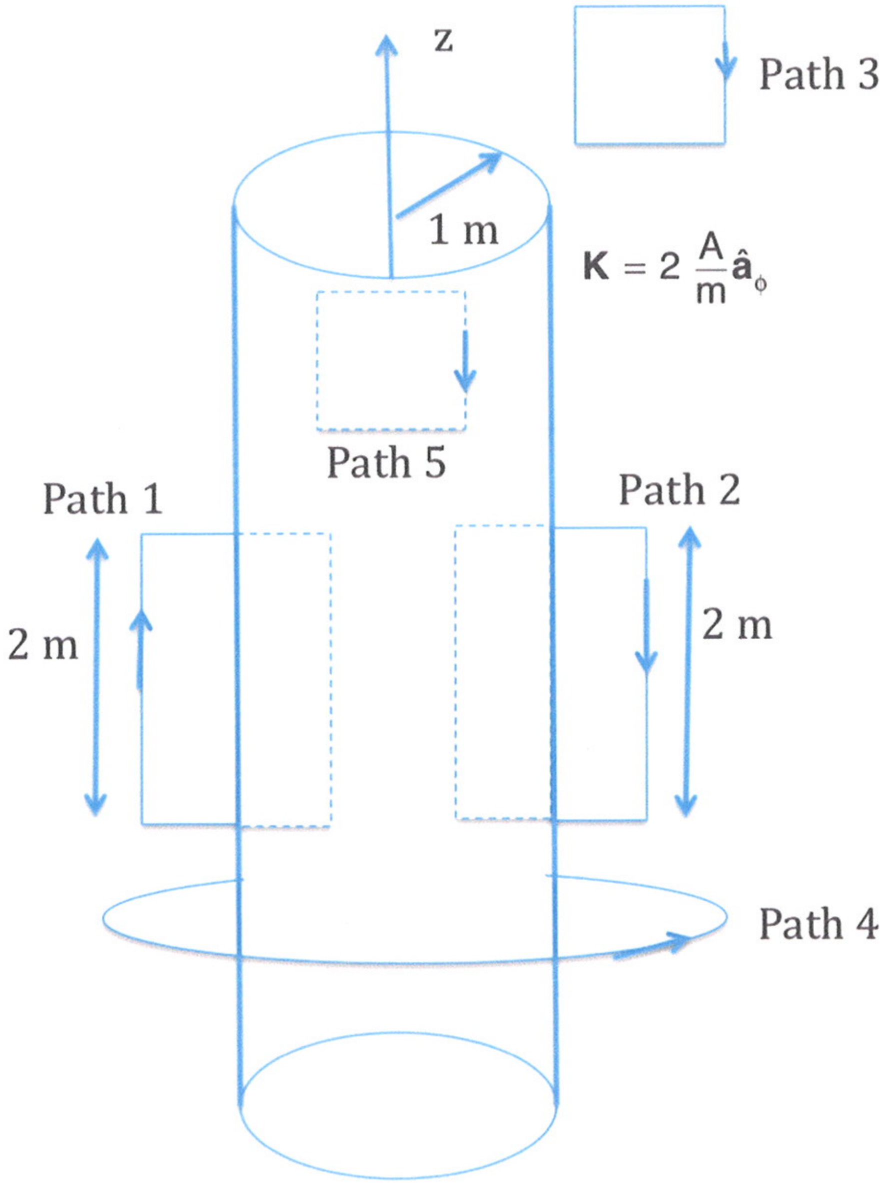
$$V(\rho) = A \ln \rho$$

$$V(2) = A \ln 2 = 100\text{V}$$

$$A = \frac{100\text{V}}{\ln 2}$$

$$V(\rho) = \frac{100\text{V}}{\ln 2} \ln \rho$$

(10 pts) 7. A sheet current density of $\mathbf{K}=2\frac{\mathrm{A}}{\mathrm{m}}\hat{\mathbf{a}}_\phi$ is flowing on the cylinder shown centered on the z-axis. For path 1 and 2 the solid line is outside the cylinder and the dashed line inside the cylinder. Path 3 is completely outside the cylinder, path 4 completely surrounds the cylinder, and path 5 is completely inside the cylinder, and.



$\oint_1 \mathbf{H}\bullet d\mathbf{l} =$ −4A

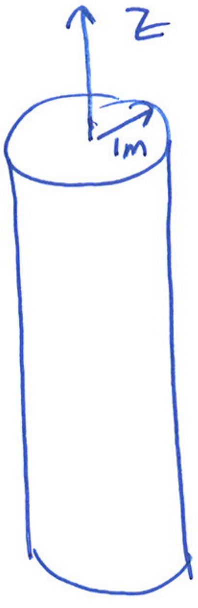
$\oint_2 \mathbf{H}\bullet d\mathbf{l} =$ +4A

$\oint_3 \mathbf{H}\bullet d\mathbf{l} =$ 0

$\oint_4 \mathbf{H}\bullet d\mathbf{l} =$ 0

$\oint_5 \mathbf{H}\bullet d\mathbf{l} =$ 0

(10 pts) 8. Determine the magnetic field intensity everywhere (inside and outside) of a solid conductor centered on the z-axis of radius $\rho = 1\text{m}$ with a current density of $J = 1\frac{\text{A}}{\text{m}^2}\hat{\mathbf{a}}_z$ for $\rho < 1\text{m}$.



The $\vec{H}$ field will have cylindrical symmetry and $\vec{H}$ will only vary with ρ and only have an $\hat{a}_\phi$ component.

for $\rho < 1m$

$$\oint \vec{H} \cdot \vec{dl} = (\pi \rho^2)\left(1\frac{A}{m^2}\right)$$

$$H_\phi \, 2\pi\rho = (\pi \rho^2)\left(1\frac{A}{m^2}\right)$$

$$\boxed{\vec{H}_\phi = \frac{\rho}{2}\frac{A}{m^2}\hat{a}_\phi \quad \text{for } \rho < 1m}$$

for $\rho > 1m$

$$\oint \vec{H} \cdot \vec{dl} = (\pi)(1m)^2\left(1\frac{A}{m^2}\right) = \pi A$$

$$H_\phi \, 2\pi\rho = \pi \; A$$

$$\boxed{\vec{H}_\phi = \frac{1}{2\rho} A \, \hat{a}_\phi \quad \text{for } \rho > 1m}$$

(10 pts) 9. Indicate whether the following statements are true or false.

The value of capacitance of a conducting sphere is dependent on the amount of charge on the sphere	True	[✓ False]
The dielectric strength is the maximum electric field that a dielectric can tolerate or withstand without electric breakdown.	[✓ True]	False
Lines of magnetic flux never terminate on themselves	True	[✓ False]
In a linear dielectric the polarization field varies linearly with the electric field intensity.	[✓ True]	False
If the current is doubled flowing through a wire coil, everywhere the magnetic field intensity is doubled.	[✓ True]	False

(12 pts).10. Fill in the table with the standard units for the following

magnetic flux density, **B**	$\frac{Wb}{m^2} = T$
Magnetic field intensity, **H**	$\frac{A}{m}$
Electric Field Intensity, **E**	$\frac{V}{m}$
Electric Flux Density, **D**	$\frac{C}{m^2}$
Electric flux, **Ψ**	C
Magnetic flux, **Ψ**	Wb